Can Underemployment Persist in an Expanding Economy? Clues from a Non-Walrasian OLG Model with Endogenous Longevity

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This paper aims at casting a new light on the persistence of underemployment in emerging economies, by examining the relationship between labour market imperfections and longevity changes. For that purpose, we develop a two-period OLG model where longevity depends positively on the real wage, but negatively on the underemployment level, which both result from wage negotiations between a trade-union, representing workers (i.e. young generation), and the management, representing capital-holders (i.e. old generation). The issues of existence, uniqueness and stability of a non-trivial steady-state equilibrium are examined. It is shown that the distribution of bargaining power in the economy is a major determinant of the short run and long run dynamics of production, employment and longevity.

Keywords: bargaining power, longevity, OLG model, underemployment, wage-bargaining.

JEL codes: I12, J52, O41.

1 Introduction

Although this phenomenon is hard to measure with accuracy, there is little doubt that a significant underemployment - i.e. underutilization of labour forces in the production process - exists nowadays in many emerging economies, and, in particular, in East-European transition economies (see Rutkowski, 2006).[1] A major cause of underemployment in developing economies has to do with the absence - or ineffectiveness - of unemployment insurance systems, which makes all people work, simply because no one can afford to be unemployed in those economies.[2]

While underemployment - and the associated 'hidden' unemployment - is typical of poor countries with a low-skilled labour force and little productivity gains, a natural object of study is the issue of its *persistence* over time. Is underemployment a transitory phenomenon, which will disappear once economies develop, or, on the contrary, can underemployment persist in expanding economies?

The answer to that question depends on the relationship between, on the one hand, underemployment, and, on the other hand, the level of economic development. That relationship is *bidirectional*: whereas a large underemployment can prevent economic expansion, a stagnant economy, by preventing any progress or reform, can also make underemployment persist (see Bowden *et al*, 2006).

Undoubtedly, the difficulty to deal with the persistence issue is due to the *multidimensionality* of the relationship between underemployment and economic development. Among others, an important dimension is the education: given that underemployment occurs generally in economies with a low-skilled labour force, a natural way to make underemployment disappear is through massive investments in education. Another crucial dimension is the institutional dimension. If underemployment is due to the imperfect functioning of the labour market, a government

[1] On the evolution of labour market in socialist and transition economies, see Kornai (2000).

[2] Note that underemployment is also present in advanced economies (see Wilkins, 2004), where unemployment insurance exists. This points out to other determinants of underemployment.

can reduce underemployment by making imperfections of labour markets disappear.

The goal of this paper is to cast a new light on the underemployment persistence issue, by exploring another dimension of the development-underemployment relationship: its *demographic* dimension. In particular, we shall pay here a particular attention to a major channel through which economic development and underemployment can influence each others: the length of human life.

In order to see why human longevity occupies a central position between economic development and underemployment, let us say a few words about the - bidirectional - relationships existing between, on the one hand, longevity and development, and, on the other hand, between longevity and underemployment.

Actually, as this was emphasized by OLG growth models with endogenous longevity (see Chakraborty, 2004; Battacharya and Qiao, 2005; Pestieau *et al,* 2006), longevity is not only influenced by the level of economic development, but it tends also, by its impact on the temporal horizon of agents, to affect capital accumulation decisions, and, hence, the whole development process.

Moreover, while demographers, such as Mesrine (2001) and Nylen *et al* (2001), showed that being constrained by the voluntary exchange condition on the labour market (e.g. being unemployed or involuntary part-timer) deteriorates longevity, one can also expect longevity to affect, by its impact on the horizons of agents acting on the labour market, the levels of wages and underemployment.[3]

In order to study the demographic dimension of the relationship between underemployment and economic development, this paper presents a two-period non-Walrasian OLG model in which longevity is influenced positively by the real wage, but negatively by (first-period) underemployment, and where underemployment can occur as the outcome of wage negotiations between a trade-union, represent-

[3]The unemployed and the involuntary part-timer are victims of the voluntary exchange condition on the labour market, because they would like to work more, but cannot force the firm to make them work more, because the firm has a veto right, as on any market transaction.

ing workers (young individuals), and the management, representing capital-holders (old individuals).[4] Given that longevity influences, under additive lifetime welfare, the (remaining) welfare of (old) capital-holders *to a larger extent* than the one of (young) workers, wage-negotiations are not independent from longevity, so that, in the present model, longevity and underemployment tend to affect each other.

The goal of this paper is to re-examine the issue of the persistence of underemployment, by studying, in the light of that non-Walrasian OLG economy where underemployment and longevity are jointly determined, the short run and long run dynamics of capital accumulation, employment and longevity.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 discusses the existence, uniqueness and stability of a steady-state. A comparative statics exercise is carried out in Section 4. Short run and long run dynamics are examined numerically in Section 5. Section 6 concludes.

2 The model

2.1 Environment

Let us consider a two-period OLG model. Time is discrete. Generations, who are of constant size N, are denoted by their date of birth. Members of a cohort t live a first period of unitary length, and, then, a second period of length h_{t+1} $(0 \leq h_{t+1} \leq 1)$.

Individuals are, in the first period, workers, and, in the second period, capital-holders. Workers work in the same firm, and are represented by a single trade union. The firm is run by capital-holders, represented by the management.

Longevity The length of the second period of life h_{t+1} depends on the real wage w_t and on the individual employment ratio l_t/l_t^s (l_t^s is the labour supply, and l_t is

[4] For other OLG frameworks with wage bargaining, see Devereux and Lockwood (1991) and Arnsperger and de la Croix (1993).

the actual employment) faced when being a worker, according to the relation:

$$h_{t+1} = Zw_t^{\psi}\left(\frac{l_t}{l_t^s}\right)^{\phi} \tag{1}$$

where ψ and ϕ are the elasticities of longevity with respect to the real wage and the employment rate. Z accounts for exogenous determinants of longevity ($Z > 0$).

If being constrained by the voluntary exchange condition on the labour market (i.e. undergoing excess labour supply) has no effect on longevity, ϕ equals 0, and the real wage is the unique determinant of longevity. However, if being constrained by the firm's veto right on the labour market generates a loss of self-respect, in the sense that the impossibility to work as many hours as desired makes workers feel useless, such feelings can deteriorate workers's health, and lower their longevity.[5] If this is so, underemployment, has, through the loss of self-respect it generates, a negative impact on longevity, and ϕ is strictly positive.

Production Production follows a Cobb-Douglas technology:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha} \tag{2}$$

where K_t denotes total capital stock at time t, L_t denotes the total quantity of labour used by the firm, while A is a productivity parameter.

The firm sets the employment level L_t that maximizes its profits $\Pi_t = Y_t - w_t L_t$.[6] Each capital-holder will receive some return R_t on his contribution to the total capital stock K_t, denoted by individual capital k_t, which corresponds to his savings

[5] Generally, a loss of self-respect deteriorates health and longevity through the adoption of risky behaviours (e.g. alcoholism). On this, see Vallin *et al* (2001).

[6] The price of the consumption good is fixed to 1.

from his previous period of life (when he was a worker).[7] Hence, profit per capital-holder is $R_t k_t = y_t - w_t \frac{L_t}{N}$, where y_t is the output per member of cohort t.

Moreover, given the absence of any unemployment insurance system in the economy, the labour available for workers is supposed to be divided *equally* among the working cohort, so that individual employment l_t is equal to L_t/N.

The fact that all members of a cohort face the same individual employment ratio l_t/l_t^s in their first period of life does not prevent the occurrence of a loss of self-respect, because this phenomenon relates to the *individual* goals of each member of the society, and, as such, is distinct from feelings of exclusion (which can exist only in a heterogenous cohort, unlike feelings of being useless).

Savings and labour supply Each individual has a utility function of the form:

$$\log U_t = \beta \log(c_t) + (1-\beta) h_{t+1} \log(\frac{d_{t+1}}{h_{t+1}}) - v(l_t) \tag{3}$$

where β is a preference parameter ($0 < \beta < 1$), while $v(l_t)$ is the disutility of labour, which is convex in l_t. Consumption during the first period c_t is equal to $l_t w_t - s_t$, and consumption during the second period d_{t+1} equals $R_{t+1} s_t$, where s_t is the saving of the worker, and R_{t+1} is the return on savings.

When deciding their savings and their labour supply, individuals take longevity as being independent from their choices, and equal to an expected level (i.e. the longevity of the previous cohort, h_t). Hence, the first-order conditions yield:

$$c_t = \frac{\beta}{\beta + h_t(1-\beta)} w_t l_t \qquad d_{t+1} = \frac{h_t(1-\beta)}{\beta + h_t(1-\beta)} w_t l_t R_{t+1} \qquad l_t^s = \frac{\beta + h_t(1-\beta)}{v'(l_t^s)} \tag{4}$$

[7] Capital-holders have a second period of length $0 \leq h_t \leq 1$, whereas production takes place during the entire period (of unitary length). Hence, some system of payment of capital-holders in advance must be set up to allow them to consume their whole savings income before dying.

2.2 The structure of the equilibrium

In each period, the equilibrium is determined in two successive steps.[8]

1. Firstly, a wage w_t is negotiated by the managers of the firm, representing capital-holders, and the trade union, representing workers.

2. Secondly, the firm chooses an output level. Then, transactions can take place.

In order to solve the equilibrium, we shall consider first the second stage (output setting), and, then, the first one (wage negotiation).

2.2.1 Output setting

When deciding its optimal production and employment, the firm, who takes the negotiated wage w_t as given, must satisfy two constraints: (1) the constraint imposed by the demand for the good on the good market; (2) the constraint imposed by the labour supply on the labour market.[9]

Regarding the demand constraint, it is not difficult to see, in the light of the above first-order conditions, that the total demand Y_t^d is equal to the output, so that the demand constraint faced by the firm is always met:

$$Y_t^d = Nw_t l_t \left[\frac{\beta}{\beta + h_t(1-\beta)} + \frac{h_t(1-\beta)}{\beta + h_t(1-\beta)} \right] + k_t R_t = w_t L_t + Y_t - w_t L_t = Y_t$$

The firm faces also an employment constraint: this cannot hire more labour than the labour supplied by workers. Thus, total employment L_t cannot exceed total labour supply L_t^s, equal, under $v(l_t) = l_t^\rho$, with $\rho > 1$, to:

[8] That structure is based on Benassy (2002, chapter 5).

[9] Those two constraints are imposed by the voluntary exchange condition: no buyer is forced to buy more than what he demands, and no seller is forced to sell more than what he supplies.

$$L_t^s = Nl_t^s = N\left(\frac{\beta + h_t(1-\beta)}{\rho}\right)^{\frac{1}{\rho}} \tag{5}$$

Hence, the problem of the firm is to select the employment L_t maximizing:

$$\begin{aligned}
\Pi_t &= Y_t - w_t L_t \\
s.t. Y_t &= AK_t^{\alpha} L_t^{1-\alpha} \\
L_t &\leq L_t^s
\end{aligned}$$

There exist two kinds of solution to that problem, depending on whether there is excess supply or excess demand on the labour market, that is, depending on whether the employment L_t is strictly smaller than labour supply L_t^s or not.

If there is excess supply on the labour market, employment is determined by the firm's demand, according to the rule 'marginal revenue equals marginal cost': $A(1-\alpha)K_t^{\alpha}L_t^{-\alpha} = w_t$. Multiplying by L_t yields $(1-\alpha)Y_t = L_t w_t$. Hence:

$$L_t = L_t^d = \left(\frac{A(1-\alpha)K_t^{\alpha}}{w_t}\right)^{\frac{1}{\alpha}} \tag{6}$$

so that the employment of each worker is equal to $l_t^d = \left(\frac{A(1-\alpha)k_t^{\alpha}}{w_t}\right)^{\frac{1}{\alpha}}$.

If there is excess demand on the labour market, the employment level is determined by workers's labour supply:

$$L_t = L_t^s = N\left(\frac{\beta + h_t(1-\beta)}{\rho}\right)^{\frac{1}{\rho}} \tag{7}$$

Hence, the wage at which labour demand equals labour supply is:

$$w_t^* = A(1-\alpha)k_t^\alpha \left(\frac{\beta + h_t(1-\beta)}{\rho}\right)^{\frac{-\alpha}{\rho}} \tag{8}$$

If the wage is higher than the Walrasian wage w_t^*, there is excess labour supply, whereas, if the wage is below w_t^*, there is excess labour demand.

2.2.2 Wage negotiations

The wage is determined by negotiations between the management of the firm, representing capital-holders, and the trade union, representing workers.

We shall assume here that the negotiated wage is equal to the asymmetric Nash bargaining solution (see Binmore *et al*, 1986), which corresponds to the maximum of the quantity Δ_t defined as:[10]

$$\log \Delta_t = \delta \log \left(U_t^w - \bar{U}_t^w\right) + (1-\delta) \log \left(U_t^c - \bar{U}_t^c\right) \tag{9}$$

where δ is the bargaining power of workers, while $1-\delta$ is the bargaining power of capital-holders.[11] U_t^w and U_t^c denote the utility of the trade-union and the management if some agreement is reached, whereas $\bar{U}_t^w$ and $\bar{U}_t^c$ correspond to the utility of the trade-union and the management in the case of no agreement.

The trade-union pursues the goal of maximizing the lifetime utility of workers, whereas the management pursues the goal of maximizing the remaining lifetime

[10] As shown in Binmore *et al* (1986), the asymmetric Nash bargaining solution can be regarded as the solution of a noncooperative sequential bargaining process, provided both parties react very quickly to each other's proposal.

[11] As this is stressed in Binmore *et al* (1986), the bargaining power parameter should not be regarded as reflecting asymmetries in the preferences of parties (which are captured by U^w and U^c), or in the disagreement points (which are modelled as $\bar{U}^w$ and $\bar{U}^c$), but, rather, as resulting from asymmetries either in the bargaining procedure (e.g. different time lags before reactions), or in the beliefs about a risk of breakdown of negotiations.

utility of capital-holders (i.e. during their second period of life).

For workers, if there is no agreement, first-period and second-period consumptions are zero, so that $\bar{U}_t^w = 0$. However, if an agreement is found:

$$\log(U_t^w) = \beta \log\left(\frac{\beta l_t w_t}{\beta + h_{t+1}(1-\beta)}\right) + (1-\beta)h_{t+1}\log\left(\frac{(1-\beta)l_t w_t R_{t+1}}{\beta + h_{t+1}(1-\beta)}\right) - v(l_t)$$

Given that h_{t+1} and R_{t+1} are taken as given (i.e. $h_{t+1} = h_t$, $R_{t+1} = \bar{R}$), $\log\left(U_t^w - \bar{U}_t^w\right)$ can be written, up to an additive constant, as:

$$\log\left(U_t^w - \bar{U}_t^w\right) = \beta \log(l_t w_t) + (1-\beta)h_t \log(l_t w_t) - v(l_t) \tag{10}$$

Longevity makes the quest for higher wages more desirable *ceteris paribus*.

Regarding capital-holders, if there is some agreement, total consumption is $d_t = k_t R_t$, whereas, if there is no agreement, production does not take place and capital becomes useless, so that d_t is zero, implying $\bar{U}_t^c = 0$. Hence,

$$\log\left(U_t^c - \bar{U}_t^c\right) = (1-\beta)h_t \log\left(\frac{k_t R_t}{h_t}\right)$$

Given that h_t and k_t are given from the previous period, and that $k_t R_t = y_t - w_t l_t$, $\log\left(U_t^c - \bar{U}_t^c\right)$ can be written, up to an additive constant, as:

$$\log\left(U_t^c - \bar{U}_t^c\right) = (1-\beta)h_t \log\left(y_t - w_t l_t\right) \tag{11}$$

Note that if h_t is low, there is little utility to be gained from profits. However, when h_t is higher, making profits becomes much more important for capital-holders. Longevity is thus a major determinant of the welfare associated to a given amount of profit, and, as such, it constitutes a central aspect of capital-holders's tenacity when bargaining with the trade-union.

While longevity plays also a similar role for workers - because a higher wage is also more welcome if longevity is larger - the role of h_t is far less important for workers: workers will, in any case, live their first period, so that, even though h_t tends to zero, workers still need a high wage. This is not the case for capital-holders: if h_t tends to zero, capital-holders have nothing to gain from wage negotiations, so that it is *as if* only workers are making the decision. Therefore, longevity affects the wage bargaining in an asymmetric manner.[12]

Let us now solve the bargaining problem. Under the assumptions made, w_t can be regarded as determined by the maximization of:

$$\log \Delta_t = \delta\beta \log(w_t l_t) + \delta(1-\beta)h_t \log(w_t l_t) - \delta v(l_t) + (1-\delta)(1-\beta)h_t \log(y_t - w_t l_t) \tag{12}$$

We can now consider the two possible outcomes of the negotiation, that are, respectively, excess labour supply and excess labour demand.

Under excess labour supply, $l_t = l_t^d = \left(\frac{A(1-\alpha)k_t^\alpha}{w_t}\right)^{\frac{1}{\alpha}}$. Hence,

$$\frac{\partial \log \Delta_t}{\partial \log w_t} = \frac{1}{\alpha}\left(\delta l_t v'(l_t) - \delta\beta(1-\alpha) - (1-\beta)(1-\alpha)h_t\right) \tag{13}$$

[12] The asymmetric influence of longevity on the utility of the young and the old - and the resulting asymmetric impact of longevity on the agreement reached - follows from the functional form representing lifetime welfare, and might not hold for other functional forms. However, other forms would yield the same outcome, provided these reflect the fact that a given longevity gain is not regarded in an equivalent way by a young and an old individual.

Equalizing that expression to zero yields: $l_t v'(l_t) = \frac{\delta\beta(1-\alpha)+(1-\beta)(1-\alpha)h_t}{\delta}$, so that it is possible to rewrite l_t as: $l_t = \Gamma\left(\frac{\delta\beta(1-\alpha)+(1-\beta)(1-\alpha)h_t}{\delta}\right)$, where $\Gamma' > 0$. This suggests that the higher δ is, the lower employment will be.

To derive the condition under which excess of labour supply prevails, it suffices to notice that optimal individual labour supply is given by: $l_t v'(l_t) = \beta + h_t(1-\beta)$. Hence, there is excess labour supply if the optimal labour supply is superior to the labour demand resulting from the negotiated wage, that is, if:

$$\delta > \frac{(1-\alpha)(1-\beta)h_t}{\alpha\beta + (1-\beta)h_t} \tag{14}$$

Thus, if that condition holds, some underemployment exists. Underemployment is more likely the higher α is, and the lower the length of second period h_t is. Indeed, if h_t equals 0, that condition collapses to $\delta > 0$, whereas, under h_t equal to 1, that condition collapses to $\delta > \frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)} > 0$, which is a stronger condition. Hence, a higher longevity makes excess labour supply less likely. In the light of the asymmetric role of longevity as a determinant of workers and capital-holders's welfares, this is not a surprise: when longevity is higher, the negotiated wage becomes more important for capital-holders than for workers. Thus, the higher h_t is, the larger the (relative) tenacity of capital-holders in wage negotiations is, and thus the lower the resulting wage is, so that, under a higher h_t, excess labour supply is, *ceteris paribus*, less likely to occur than under a lower h_t.

The negotiated wage is equal to:

$$w_t = A(1-\alpha)k_t^{\alpha}\left(\frac{(1-\alpha)\left[\delta\beta + (1-\beta)h_t\right]}{\delta\rho}\right)^{\frac{-\alpha}{\rho}} \tag{15}$$

If the above condition is satisfied, that negociated wage w_t exceeds the Walrasian wage w_t^*, and the economy lies in a situation of excess labour supply.

While underemployment can be regarded, at the level of the union, as *voluntary* (because this results from the union's attitude in the bargaining), this may be nonetheless considered as *involuntary* at the level of workers, in the sense that it may reflect union's incapacity to represent workers perfectly.[13]

Under excess labour demand, $l_t = l_t^s = \left(\frac{\beta+(1-\beta)h_t}{\rho}\right)^{\frac{1}{\rho}}$, so that:

$$\frac{\partial \log \Delta_t}{\partial \log w_t} = \delta\beta + \delta(1-\beta)h_t - (1-\delta)(1-\beta)h_t \frac{w_t l_t}{y_t - w_t l_t} \tag{16}$$

Equalizing that condition to zero yields:

$$w_t = \frac{\delta y_t \left(\beta + (1-\beta)h_t\right)}{(\delta\beta + (1-\beta)h_t)l_t} \tag{17}$$

Differentiating this with respect to h_t shows that a higher longevity is likely to lead to a lower wage, except if δ is equal to 1, or if β is equal to 1. Otherwise, if capital-holders have some power, the negotiated wage is - although there is excess labour demand - strictly *decreasing* with longevity, because of the asymmetric role of longevity as a determinant of capital-holders and workers's welfares.

There is excess labour demand if the derivative $\frac{\partial \log \Delta_t}{\partial \log w_t}$ is negative at the wage w_t^* at which labour supply equals labour demand. This yields the condition:

$$\delta < \frac{(1-\alpha)(1-\beta)h_t}{\alpha\beta + (1-\beta)h_t} \tag{18}$$

[13] Actually, unions do not internalize the impact of wage negotiations on longevity, and, thus, the wage resulting from negotiations may not correspond to what a well-informed, fully-rational union would have negotiated.

The higher h_t is, the more likely excess labour demand is. The negotiated wage is:

$$w_t = Ak^{\alpha}\frac{\delta\rho^{\frac{\alpha}{\rho}}\left(\beta+(1-\beta)h_t\right)^{\frac{\rho-\alpha}{\rho}}}{(\delta\beta+(1-\beta)h_t)} \tag{19}$$

Under $\delta < \frac{(1-\alpha)(1-\beta)h_t}{\alpha\beta+(1-\beta)h_t}$, the negociated wage is strictly lower than the Walrasian wage w_t^*, so that the economy lies in a situation of excess labour demand.

3 Equilibria

Let us now characterize a temporary equilibrium in our economy, and discuss the existence, uniqueness and stability of a non-trivial steady-state equilibrium.

3.1 Temporary equilibria

Given variables from the previous period $\left\{s_{t-1}, w_{t-1}, l_{t-1}, l_{t-1}^s\right\}$, we can now characterize the temporary equilibrium $\{k_t, h_t, l_t, y_t, w_t, R_t, c_t, s_t, d_t\}$ as follows:

$$\begin{aligned}
k_t &= s_{t-1}; h_t = Zw_{t-1}^{\psi}\left(\frac{l_{t-1}}{l_{t-1}^s}\right)^{\phi}; l_t = \min\left(\left(\frac{\delta\beta+(1-\beta)h_t}{(1-\alpha)^{-1}\delta\rho}\right)^{\frac{1}{\rho}}, \left(\frac{\beta+(1-\beta)h_t}{\rho}\right)^{\frac{1}{\rho}}\right)\\
y_t &= \frac{Y_t}{N} = \frac{AK^{\alpha}L^{1-\alpha}}{N^{\alpha}N^{1-\alpha}} = Ak_t^{\alpha}l_t^{1-\alpha}\\
w_t &= A(1-\alpha)k_t^{\alpha}\min\left(\left(\frac{\delta\beta+(1-\beta)h_t}{(1-\alpha)^{-1}\delta\rho}\right)^{\frac{-\alpha}{\rho}}, \frac{\delta\rho^{\frac{\alpha}{\rho}}\left(\beta+(1-\beta)h_t\right)^{\frac{\rho-\alpha}{\rho}}}{(1-\alpha)(\delta\beta+(1-\beta)h_t)}\right)\\
R_t &= \frac{AK_t^{\alpha}L_t^{1-\alpha}-w_tL_t}{K_t} = \frac{Ak_t^{\alpha}l_t^{1-\alpha}-w_tl_t}{k_t}\\
c_t &= \frac{\beta}{\beta+h_t(1-\beta)}w_tl_t; s_t = \frac{h_t(1-\beta)}{\beta+h_t(1-\beta)}w_tl_t; d_t = R_ts_{t-1}
\end{aligned}$$

The variables l_t and w_t being determined by h_t by means of single-valued functions $l(h_t)$ and $w(h_t)$, that system defines a unique temporary equilibrium.

That temporary equilibrium can be either an equilibrium at which there is excess labour supply, or excess labour demand, depending on whether:

$$\delta \begin{array}{c} > \\ = \\ < \end{array} \frac{(1-\alpha)(1-\beta)h_t}{\alpha\beta + (1-\beta)h_t}$$

That condition depends on h_t, as well as on parameters α, β and δ.

3.2 Inter-temporal equilibria

3.2.1 Existence

Regarding the existence of a steady-state equilibrium, the above system suggests that, if h_t is constant, individual employment l_t and individual labour supply l_t^s are constant, so that underemployment is constant. Hence, if k_t and h_t are constant, all variables are constant. Thus, a steady-state equilibrium requires two conditions:

$$\begin{aligned} k_{t+1} &= \frac{h_t(1-\beta)}{\beta + h_t(1-\beta)} w(k_t, h_t) l_t(h_t) = k_t \\ h_{t+1} &= Z \left(w(k_t, h_t) \right)^{\psi} \left(e(h_t) \right)^{\phi} = h_t \end{aligned}$$

where $e(h_t)$ denotes the employment function, defined as $e(h_t) = \min \left(\frac{l_t(h_t)}{l_t^s(h_t)}, 1 \right)$.

Let us now define the kk locus, as the set of points in the (k, h) space along which k_t is constant:

$$k = \left(\frac{A(1-\alpha)(1-\beta)h}{\beta + (1-\beta)h} \Theta \right)^{\frac{1}{1-\alpha}} \Xi^{\frac{1}{1-\alpha}} \tag{20}$$

where $\Theta \equiv \min \left(\left(\frac{\delta\beta + (1-\beta)h}{(1-\alpha)^{-1}\delta\rho} \right)^{\frac{-\alpha}{\rho}}, \frac{\delta\rho^{\frac{\alpha}{\rho}} (\beta + (1-\beta)h)^{\frac{\rho-\alpha}{\rho}}}{(1-\alpha)(\delta\beta + (1-\beta)h)} \right)$, $\Xi \equiv \min \left(\left(\frac{\delta\beta + (1-\beta)h}{(1-\alpha)^{-1}\delta\rho} \right)^{\frac{1}{\rho}}, \left(\frac{\beta + (1-\beta)h}{\rho} \right)^{\frac{1}{\rho}} \right)$.

If, for a given h level, k is higher than this level, k must fall. On the contrary, if, for a given h level, k is lower than the above level, k must grow.

Similarly, the hh locus is defined as follows:

$$k = \left[\frac{Z}{h} \left(A(1-\alpha)\Theta \right)^{\psi} \right]^{\frac{-1}{\alpha\psi}} \left[\min \left(\frac{\left(\frac{(1-\alpha)[\delta\beta + (1-\beta)h]}{\delta\rho} \right)^{\frac{1}{\rho}}}{\left(\frac{\beta + (1-\beta)h}{\rho} \right)^{\frac{1}{\rho}}}, 1 \right) \right]^{\frac{-\phi}{\alpha\psi}} \tag{21}$$

For a given h, a capital stock exceeding this critical level leads to a rise in h, whereas a capital stock lower than it leads to a fall in h.

Actually, the question of the existence of a steady-state in the economy can be formulated as whether the kk locus and hh locus intersect. Whether the steady-state is characterized by some underemployment or not depends on whether the two loci intersect in an area of the (h, k) space where longevity is inferior or superior to the level associated to the above underemployment condition.

As we shall now see, various cases may arise, depending on whether a steady-state exists or not, and, if yes, on its characteristics (underemployment or not).

Figure 1 shows that, under $\delta = 0.1$, $\alpha = 0.5$ and $\beta = 0.5$, the two loci intersect in an area of the (h, k) space characterized by full employment. Given the values of α, β and δ, it is when h equals 0.125 that labour demand equals labour supply. Hence, all temporary equilibria on the left of the vertical dotted line (at $h = 0.125$) are underemployment equilibria, whereas all equilibria on the right are full employment equilibria. The steady-state is here a full employment equilibrium.[14]

[Place Figure 1 around here]

Whereas Figure 1 showed the existence of a full employment equilibrium under a low δ, such a low bargaining power of workers does not guarantee a full employment

[14] On Figure 1, $A = 20, \alpha = 0.5, \beta = 0.5, \delta = 0.1, \psi = 0.2, \phi = 0.1, Z = 0.2$.

steady-state. As shown on Figure 2, bad exogenous survival conditions (a lower parameter Z) can, despite the low δ, lead to an underemployment steady-state.[15]

Let us now consider the case where workers have a bigger bargaining power. It follows from the condition underlined in Section 2 that, if α is fixed to $1/2$ and β to $1/2$, then, for any value of $h \in [0,1]$, excess labour supply will prevail for values of δ higher than $1/3$. Thus, if one supposes that δ is $1/2$, the steady-state, if it exists, must be characterized by underemployment, as on Figure 3.[16]

[Place Figures 2, 3 and 4 around here]

However, while this model accounts for the existence of full employment and underemployment steady-states, the existence of such non-trivial equilibria is not always garanteed, as shown by Figure 4, where zero is the unique steady-state.[17]

The possibility of non-existence of a non-trivial steady-state raises the question of the conditions that garantee the existence of such an equilibrium.

Proposition 1 *In the case where* $\delta > \frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$*, the following conditions suffice to garantee the existence of a non-trivial underemployment steady-state:*

$$Z\left[A(1-\alpha)\right]^{\frac{\psi}{1-\alpha}}(1-\beta)^{\frac{\alpha\psi}{1-\alpha}}\left(\frac{\delta\beta+(1-\beta)}{(1-\alpha)^{-1}\delta}\right)^{\frac{\phi}{\rho}}<1 \quad ; \quad \frac{\alpha\psi}{1-\alpha}<1$$

Proposition 2 *In the case where* $0<\delta\leq\frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$*, the following conditions suffice to garantee the existence of a non-trivial steady-state, which can be characterized either by some underemployment or by full employment:*

$$Z\left[A(1-\alpha)\right]^{\frac{\psi}{1-\alpha}}(1-\beta)^{\frac{\alpha\psi}{1-\alpha}}\left(\frac{\delta\beta+(1-\beta)}{(1-\alpha)^{-1}\delta}\right)^{\frac{-\psi}{1-\alpha}}<1 \quad ; \quad \frac{\alpha\psi}{1-\alpha}<1$$

[15] On Figure 2, $A=20, \alpha=0.5, \beta=0.5, \delta=0.1, \psi=0.2, \phi=0.1, Z=0.05$.
[16] On Figure 3, $A=20, \alpha=0.5, \beta=0.5, \delta=0.5, \psi=0.2, \phi=0.1, Z=0.2$.
[17] In Figure 4, $A=20, \alpha=0.7, \beta=0.5, \delta=0.5, Z=0.02, \psi=0.3, \phi=0.2$.

Proposition 3 *In the case where $\delta = 0$, there exists a unique steady-state:* $(0, 0)$.

The intuition behind those results goes as follows.[18]

In the case where workers have a strong bargaining power $\delta > \frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$, two conditions garantee the existence of a non-trivial steady-state. The first, trivial, condition is that, when longevity h_t tends to its upper bound (equal to 1), the laws of demography and economy are such that h_{t+1} remains within the interval $[0, 1]$ on which longevity is defined.[19] The second condition, which imposes that $\frac{\alpha\psi}{1-\alpha}$ must be strictly lower than 1, imposes a restriction on the importance of capital in the production process, and on the elasticity of longevity with respect to the real wage. That condition is quite weak: given that ψ is likely to be low, the condition may be satisfied even when α takes high values. Those conditions remain almost unchanged when δ is allowed to take a positive value inferior to $\frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$. Finally, if trade-unions have no bargaining power ($\delta = 0$), it is obvious, given that capital is accumulated through workers' savings, that the economy must necessarily collapse.

3.2.2 Uniqueness

We are now in position to discuss the uniqueness of a non-trivial steady-state. For that purpose, we shall first consider the uniqueness of a full employment steady-state, and, then, the uniqueness of an underemployment steady-state.

Regarding the uniqueness of a steady-state at which there exists no excess labour supply, it is easy to see that such an equilibrium, if it exists, must necessarily be the unique steady-state with that property, as stated below.[20]

Proposition 4 *If it exists, a (non-trivial) full employment steady-state must be the unique full employment steady-state.*

[18] The proofs of those claims are presented in the appendix.

[19] Thus, that condition states that the maximum longevity cannot be overtaken, which is a quite tautological requirement.

[20] A proof of that statement is presented in the Appendix.

Let us give some intuitions behind that result. As illustrated by Figure 5, which shows the full employment steady-state under the calibration of Figure 1, the function $h_{t+1} = G(h_t)$, obtained by substituting the kk locus within the hh locus, is *decreasing* once underemployment has disappeared (i.e. once $h_t > 0.125$).[21] Thus, $G(h_t)$ can no longer intersect the 45° line once full employment prevails.[22]

[Place Figure 5 around here]

The negative slope of $G(h_t)$ under full employment is due to the fact that, under full employment, h_{t+1} depends only on w_t, which affects it positively. However, h_t has, *ceteris paribus*, a negative impact on the negotiated wage w_t, so that, under full employment, h_t and h_{t+1} are negatively related to each others, as reflected by the negative slope of $G(h_t)$ on the right of the vertical dotted line. Thus, when it exists, a full employment steady-state must be the unique full employment steady-state.

However, does uniqueness also prevail when an underemployment steady-state exists? Actually, an underemployment steady-state is not necessarily unique: three cases may occur, depending on the parameters α, δ, ψ and ϕ.

Firstly, the underemployment steady-state, if it exists, may be unique, as on Figure 6.[23] Alternatively, it may be the case, as on Figure 7, that an underemployment steady-state (on the left of the vertical dotted line) coexists with a full employment steady-state (on the right of the vertical dotted line).[24] The possibility of coexistence of two different kinds of steady-states does not conflict with Proposition 4: while Proposition 4 was based on the property of a negative derivative $G'(h)$ in the full employment area, that property does not prevent the function $G(h)$ from crossing the 45° line in the underemployment area.

[21] $G(h_t)$ gives us the value of h_{t+1} as a function of h_t *provided* k_t remains constant.
[22] On Figure 5, $A = 20, \alpha = 0.5, \beta = 0.5, \psi = 0.2, \phi = 0.1, Z = 0.2$.
[23] On Figure 6, $A = 20, \alpha = 0.5, \beta = 0.5, \delta = 0.5, \psi = 0.2, \phi = 0.1, Z = 0.2$.
[24] On Figure 7, $A = 20, \alpha = 0.8, \beta = 0.5, \delta = 0.1, \psi = 0.42, \phi = 0.85, Z = 0.2$.

[Place Figures 6, 7 and 8 around here]

Finally, two underemployment steady-states may also coexist, as on Figure 8.[25] Here, the bargaining power of trade-unions is fixed to such a high level (i.e. $\delta = 0.9$) that no full employment steady-state exists. Nonetheless, there exist two distinct underemployment steady-states, one taking place a low longevity level, whereas the other occurs at a high longevity level.

3.2.3 Stability

Let us now consider the stability of steady-states graphically.[26] $(0, 0)$ being always a steady-state, we shall concentrate here on the *local* stability of non-trivial steady-states. Here again, several cases should be distinguished.

In the case where there exists a non-trivial full employment steady-state, such an equilibrium must be unique. As illustrated by Figure 1bis, a full employment steady-state is likely to be locally stable: whatever the initial capital and longevity are, there must be a convergence towards the steady-state.

[Place Figure 1bis around here]

On the left of the hh locus, longevity must grow, whereas, on the right of it, h must fall. Hence, if the initial longevity level is, for instance, 0.2, longevity will grow. Two cases can arise, depending on the initial level of capital: if it lies below the kk locus (as on Figure 1bis, where $k_0 = 0.5$), both capital and longevity will grow towards the steady-state; if, on the contrary, k_0 lies above the kk locus, there will be, in a first stage, a rise in h and a fall in k until the economy has entered the area between the two loci, where both h and k grow.

[25] On Figure 8, $A = 25, \alpha = 0.9, \beta = 0.5, \delta = 0.9, \psi = 0.15, \phi = 0.14, Z = 0.75$.

[26] Given the discontinuity of the wage function, we confine ourselves here to a non-formal study of stability, whose conclusions cannot have the generality of the ones of a formal study.

While a full employment steady-state, when it is the unique steady-state, is likely to be locally stable, one should notice that, when it coexists with an underemployment steady-state, the full employment steady-state is, unlike the other equilibrium, locally stable. To see this, let us turn back to Figure 7, and consider the initial situation ($k_0 = 6, h_0 = 0.7$). As illustrated on Figure 7bis (showing the first 10 temporary equilibria), the economy will move upwards in the area between the loci, and will converge towards the full employment steady-state.

The unstability of the low steady-state appears also clearly: for a slightly higher longevity level, the economy, being in the area at the left of the steady-state, between the kk and the hh loci, will be pushed away towards the high steady-state; on the contrary, for a slightly lower longevity level, the economy, which lies in an area where both longevity and capital must fall, will converge towards the origin of axis.

Although Figure 7bis shows an unstable underemployment steady-state, unstability does not always prevail for that kind of equilibrium. Actually, there can exist a locally stable steady-state with underemployment in two cases: firstly, when this is unique; and, secondly, when there exist several underemployment steady-states. We shall here concentrate on the latter case.

[Place Figures 7bis and 8bis around here]

Drawing the horizontal and vertical arrows on Figure 8bis, based on Figure 8, shows that the higher underemployment steady-state is locally stable, whereas the lower underemployment steady-state is unstable. To illustrate the local stability of the high underemployment equilibrium, we computed the first 15 temporary equilibria starting from ($k_0 = 1.5$, $h_0 = 0.80$). Thus, while an underemployment steady-state is unstable when it coexists with a full employment equilibrium, this is not necessarily true when it coexists with an underemployment steady-state.

4 Comparative statics

Let us now examine the impact, *ceteris paribus*, of a change of a parameter on the steady-state. For that purpose, we shall, for each parameter, consider two distinct cases: an initial full employment steady-state and an initial underemployment steady-state.

It does not come as a surprise that the bargaining power of workers δ has a strong influence on steady-state capital and longevity. As shown by Figure 9a, a rise in δ from 0.10 to 0.15, by shifting the kk locus upwards, has a positive effect on steady-state capital.[27] Moreover, this tends also here to push the hh locus towards the right, so that steady-state longevity is also increased. But the rise in workers' bargaining power does not, in this case, create underemployment: although the vertical dotted line shifts to the right with the rise of δ, the new steady-state still lies at the right of that line, so that full employment still holds under the high δ.

However, that latter result does not always hold: a rise in δ can also make underemployment appear. On Figure 9b, where δ is equal to 0.3, the new steady-state lies on the left of the new vertical dotted line. Hence, the new steady-state is no longer a full employment one. Thus, starting from a full employment steady-state, a rise in the workers' bargaining power can deteriorate employment. Similarly, if underemployment prevails initially, a rise in δ will deteriorate employment.

[place Figures 9a,b,c,d around here]

Whereas those examples suggest that the rise in δ, although it may deteriorate employment, may increase k^* and h^*, it might be the case, if underemployment affects h significantly (i.e. under a higher ϕ), that a rise in δ reduces h^*. That case is illustrated on Figure 9c, which shows the effect of a rise of δ from 0.1 to 0.3 under

[27] On Figures 9a and 9b, $A = 20, \alpha = 0.5, \beta = 0.5, \rho = 2, Z = 0.2, \psi = 0.2, \phi = 0.1$.

ψ equal to 0.1 and ϕ equal to 0.9.[28] Hence, a rise in δ can, despite its positive effect on k^*, lead to a lower h^*. Moreover, whereas the above rises of δ tended to increase k^*, this is not necessarily true. As suggested by Figure 9d, showing the impact of a rise of δ from 0.5 to 0.8, a rise in δ can shift the hh locus to the left and the kk locus to the right, so that both steady-state h *and* k lie below their initial levels.[29]

In sum, a rise in workers' bargaining power can have quite different effects on steady-state capital, longevity and employment, depending on the initial and final levels of δ: while a rise in δ can, when δ is low, enhance both k^* and h^* without deteriorating employment, it may reduce k^*, h^* and e^* when δ is high. Moreover, the impact of δ on the steady-state is influenced by demographic parameters. Given the negative relation between employment and δ, the impact of a rise in δ on k^* and h^* is more likely to be favourable the higher ψ is, and the lower ϕ is.

Let us now turn to the impact of demographic parameters ψ and ϕ. As shown by Figure 10a, a rise in ψ leads, *ceteris paribus*, to a shift of the hh locus towards the right, which implies higher steady-state capital and longevity.[30] Moreover, while a higher ψ does not affect employment if the initial steady-state involved full employment, it reduces underemployment when the initial steady-state involved underemployment, as shown on Figure 10b.[31] In that latter case, the substantial rise in ψ leads to a full employment steady-state.

Naturally, a change in ϕ affects the steady-state only if underemployment prevailed initially. Figure 10c shows that, if the initial equilibrium exhibits underemployment, a rise in ϕ reduces $k^*$and h^*, and, hence, deteriorates employment further.[32] On the contrary, the parameter Z, accounting for the exogenous factors

[28] On Figure 9c, $A = 5, \alpha = 0.5, \beta = 0.5, \rho = 2, Z = 0.2, \psi = 0.1, \phi = 0.9$.
[29] On Figure 9d, $A = 5, \alpha = 0.5, \beta = 0.5, \rho = 2, Z = 0.2, \psi = 0.2, \phi = 0.1$.
[30] On Figure 10a, $A = 20, \alpha = 0.5, \beta = 0.5, \delta = 0.1, \rho = 2, Z = 0.2, \phi = 0.1$. The parameter ψ takes the values $0.1, 0.2$ and 0.3.
[31] On Figure 10b, $A = 20, \alpha = 0.5, \beta = 0.5, \delta = 0.3, \rho = 2, Z = 0.2, \phi = 0.1$. The parameter ψ takes the values $0.1, 0.2$ and 0.4.
[32] On Figure 10c, $A = 20, \alpha = 0.5, \beta = 0.5, \delta = 0.3, \rho = 2, Z = 0.2, \psi = 0.2$. The parameter ϕ takes the values $0.1, 0.2$ and 0.8.

affecting longevity, has a positive influence on k^* and h^*, as well as on e^* (if some underemployment prevailled initially), as shown by Figure 10d.[33]

[Place Figures 10a,b,c,d around here]

Let us now consider the influence of preference parameters β and ρ. As shown by Figure 11a, a rise in β - implying that individuals assign a higher value to the present compared to the future - tends to push the kk locus downwards, and to push the hh locus to the right.[34] This implies a lower k^*, but has an indeterminate effect on h^*. Moreover, the impact of a higher β on e^*, although inexistant on Figure 11a, may be non-negligible: under alternative conditions, it may be the case that the new steady-state lies on the left of the new vertical dotted line (which has shifted to the right). Furthermore, if the initial equilibrium exhibited underemployment, a higher β would deteriorate employment further. As shown by Figure 11b, the parameter ρ tends to have a relatively small - almost negligible - impact on the steady-state, as long as it remains superior to unity (as supposed here).[35]

[Place Figure 11a,b around here]

Those various effects are summarized on the table below. A first observation concerns the importance of demographic parameters ψ, ϕ and Z, which influence not only steady-state longevity, but, also, steady-state capital and - provided some underemployment prevails initially - steady-state underemployment.

[Place Table 1 around here]

[33] On Figure 10d, $A = 20, \alpha = 0.5, \beta = 0.5, \delta = 0.3, \rho = 2, \psi = 0.1, \phi = 0.1$ The parameter Z takes the values $0.2, 0.3$ and 0.4.

[34] On Figure 11a, $A = 20, \alpha = 0.5, \delta = 0.3, \rho = 2, \psi = 0.1, \phi = 0.1$, $Z = 0.2$. The parameter β takes values 0.5 and 0.6.

[35] On Figure 11b, $A = 20, \alpha = 0.5, \delta = 0.3, \beta = 0.5, \psi = 0.1, \phi = 0.1$, $Z = 0.2$. The parameter ρ takes values 2 and 1.2.

Another point to be stressed is the - ambiguous - impact of workers's bargaining power on steady-state capital, longevity and employment. Thus, a higher δ is not necessarily better for workers, so that, although strictly positive, the optimal δ may be lower than 1, depending on the parameters of the model.

In sum, this comparative statics exercise highlights that the long run equilibrium depends on parameters of various natures: not only production parameters, but, also, demographic, political, and behavioural parameters.

5 Short-run and long-run dynamics

The dynamics of economies with different δ Let us first consider three economies that differ only regarding workers's bargaining power. All economies exhibit initially a longevity h_0 equal to 0.25 (this corresponds, for a working period of length 40 years, to a total length of life of 65 + 10 = 75 years), and an initial capital stock k_0 equal to 1. Moreover, α is equal to 0.35, whereas A is fixed to 20. Full depreciation is here natural given the length of a period (equal to 40 years). Regarding preferences, we shall assume that β equals 0.6, and that ρ equals 2.

Under an initial h equal to 0.25, fixing δ at 0.3 yields an initial underemployment rate of 6.9 percents. Alternatively, assuming δ equal to 0.5 and 0.7 yields an initial underemployment of respectively 13.8 and 16.9 percents.

If demographic parameters ψ, ϕ and Z are equal to respectively 0.2, 0.1 and 0.3, the steady-state longevity is equal, for δ equal to 0.3, 0.5 and 0.7, to about 0.56, which corresponds to a steady-state longevity of about 87.5 years.

Figures 12a-c illustrate the dynamics of capital accumulation, longevity and underemployment for those three economies (the economy with $\delta = 0.3$ appears in bold, the one with $\delta = 0.5$ appears in thin, and the one with $\delta = 0.7$ appears in dotted lines). Economies are represented from period 0 to period 10.

[Place Figures 12a,b,c around here]

As shown by Figure 12a, the international gap in k, which is initially small, grows over time, and is stabilized at a high level (especially if one contrasts k^* under $\delta = 0.5$ and $\delta = 0.7$). Thus, differences in δ across countries have their bigger effects in the long run. Whereas the economy with the lowest δ exhibits a higher k^* than the economy with δ equal to 0.5 up to period 5, the two curves cross each others around period 6, so that the economy with δ equal to 0.5 exhibits a higher k^* than economies where either capital-holders or workers are dominant. The relation between k^* and δ is thus non-monotonic, as we saw above.

Regarding longevity, Figure 12b suggests that there exists, at the steady-state, a small gap in favour of the economy where $\delta = 0.5$, although the gap is smaller than in terms of k^*. Thus, the relation between δ and h^* is non-monotonic: a higher bargaining power for workers does not necessarily lead to a longer life.

However, the relation between δ and steady-state underemployment is here, as shown by Figure 12c, monotonic. The gap between the economy with $\delta = 0.7$ and the one with $\delta = 0.3$, which amounts to 10 percent initially, is much larger at the steady-state. Hence, a - relatively small - international differential in δ can suffice to generate a large underemployment gap at the steady-state.

In the above example, differences in the division of bargaining power have their major impact on steady-state underemployment, and, to a smaller extent, on steady-state capital, but little effect on steady-state longevity. While this may well have been observed in actual economies, this may not necessarily be the case. To illustrate the influence of heterogeneity in δ on longevity, let us now compare two economies with a different δ (0.3 and 0.5), but where underemployment affects h more than the real wage. For that purpose, we assume $\psi = 0.1$ and $\phi = 0.2$.

[Place Figures 13a,b,c around here]

As shown by Figure 13a, k^* in the economy where $\delta = 0.3$ (solid curve) exceeds k^* in the economy where $\delta = 0.5$ (dotted curve). Thus, a larger influence of underemployment on longevity tends to reinforce the influence of δ on k^*. Figure 13b illustrates that fixing $\phi > \psi$ can lead also to a sizeable gap in terms of steady-state longevity, which is higher in the country where $\delta = 0.3$. Hence, a difference in the distribution of power can also generate significant gaps in steady-state longevity. Moreover, Figure 13c shows that underemployment will keep on prevailing at the steady-state *even* in the economy with $\delta = 0.3$, unlike what was the case under $\psi > \phi$. Hence, a strong negative impact of underemployment on longevity can not only lower h^*, but, also, can prevent the disappearance of underemployment.[36]

The convergence phenomenon Let us now compare an advanced economy and three developing economies, which have the same production function (i.e. $A = 20$, $\alpha = 0.35$), the same longevity production function (i.e. $Z = 0.3$, $\psi = 0.2$, $\phi = 0.1$), and the same preference parameters β and ρ (equal to 0.6 and 2), but differ regarding initial k and h. Whereas the advanced economy exhibits initially ($k_0 = 1$, $h_0 = 0.25$), developing economies all exhibit ($k_0 = 0.25$, $h_0 = 0.05$). While bargaining power is equally divided in the advanced economy (i.e. $\delta = 0.5$), developing economies can be of three kinds: 'quasi-dictatorship' of capital-holders ($\delta = 0.1$), of workers ($\delta = 0.9$) or symmetry ($\delta = 0.5$).

As shown by Figure 14a, the developing economy with balanced bargaining power (in thin line) reaches the same steady-state capital as the advanced economy (bold line).[37] Developing economies with δ equal to 0.1 (dotted line) and 0.9 (dashed line) do not perform as well as the balanced economy in the long run. The former exhibits no convergence at all, whereas the latter starts diverging after period 4, so

[36] That observation holds also when $\delta = 0.5$: under $\psi = 0.2$ and $\phi = 0.1$, steady-state underemployment is about 10 %, whereas it is, under $\psi = 0.1$ and $\phi = 0.2$, about 12 %.

[37] That result does not always hold, as there might exist several steady-states (see *supra*).

that there remains a large gap at the steady-state.[38]

[Place Figures 14a,b,c around here]

As far as longevity is concerned, the three developing countries exhibit a strong convergence with respect to the advanced economy during the first period, but their paths start then diverging. While the balanced economy converges to the advanced one, the economy where δ equals 0.9 tends to a slightly lower h^*, whereas the one with $\delta = 0.1$ exhibits a much lower h^*.

Regarding underemployment, convergence only occurs for the balanced economy, whereas, for the others, the prevailing underemployment rates reflect the bargaining power parameters δ, and no convergence is here to be expected: underemployment remains high where $\delta = 0.9$, and zero where $\delta = 0.1$.

To sum up, the average size of the convergence process is relatively large for longevity, but smaller for capital and underemployment. However, that conclusion in average terms must be qualified, as the distribution of bargaining power affects significantly the convergence process. In this example, developing economies where workers' bargaining power exceeds the one in the advanced economy will reach a lower steady-state (k^*, h^*, e^*), but may still exhibit some convergence in k and h, whereas economies where δ is low will remain far from the advanced economy.[39]

6 Conclusions

This paper aimed at casting a new light on the persistence of underemployment, by exploring the short run and long run dynamics of a two-period OLG economy with endogenous longevity, where the underemployment resulting from wage negotiations

[38] But this gap is less sizeable than under $\delta = 0.1$, and also smaller than the gap at $t = 0$.

[39] Note that the impact of δ on convergence is robust to the calibration of the longevity function (e.g. $\psi = 0.1 < \phi = 0.2$).

between workers and capital-holders can influence longevity. The specificity of that economy lies in the bidirectional relation between longevity and underemployment: longevity is not only affected by the real wage and by underemployment, but it does also, by its influence on workers and capital-holders's welfares, influence the outcome of wage negotiations, and, thus, the real wage and underemployment levels.

In that economy, a full employment steady-state, when it exists, must be the unique full employment steady-state, and is likely to be locally stable. Alternatively, three other cases can arise: (1) a full employment steady-state can coexist with an underemployment steady-state - the former being locally stable while the latter is unstable -; (2) two underemployment steady-states (including a locally stable one) can coexist; (3) no steady-state exists. Hence, that model allows either a short run, *transitory* underemployment (vanishing when the economy grows), or a *persisting* (steady-state) underemployment (remaining despite economic expansion).

A comparative statics exercise emphasized that the influence of workers' bargaining power on steady-state capital, longevity and employment is largely ambiguous, and depends, among other things, on the calibration of the longevity production function. Regarding the transition dynamics, it was shown numerically that economies differing only on the distribution of bargaining power exhibit quite different properties in the long run, even though the differentials are, in the short run, less sizeable. Moreover, how the bargaining power is distributed in less developed economies may have a crucial influence on the convergence phenomenon.

In sum, this study suggests that the extent to which underemployment disappears when the economy takes off depends not only on the distribution of bargaining power, but, also, on other - e.g. demographic - parameters. This paper tends thus to re-emphasize a too often neglected truth: the interconnectedness of *all* dimensions of development: not only economic, but, also, institutional and demographic.

7 References

Arnsperger C, de la Croix D (1993) Bargaining and equilibrium unemployment. Narrowing the gap between New Keynesian and 'disequilibrium' theories. European Journal of Political Economy 9: 163-190.

Benassy J-P (2002) The Macroeconomics of Imperfect Competition and Nonclearing Markets. The MIT Press.

Bhattacharya J, Qiao X (2005) Public and private expenditures on health in a growth model. Iowa State University Working Paper #05020.

Binmore K, Rubinstein A, Wolinsky A (1986) The Nash Bargaining Solution in economic modelling. The RAND Journal of Economics 17(2): 176-188.

Bowden S, Higgins DM, Price C (2006) A very peculiar practice: underemployment in Britain during the interwar years. European review of Economic History 10(1): 89-108.

Chakraborty S (2004) Endogenous lifetime and economic growth. Journal of Economic Theory 116: 119-137.

Devereux M, Lockwood B (1991) Trade unions, non-binding wage agreements, and capital accumulation. European Economic Review 35: 1411-1426.

Kornai J (2000) The Socialist System. Oxford University Press, Oxford.

Mesrine A (2001) La surmortalité des chômeurs: un effet catalyseur du chômage?. Economie et Statistiques 334(4): 33-47.

Nylén L, Voss M, Floderus B (2001) Mortality among women and men relative to unemployment, part time work, overtime work, and extra work: a study based on data from the Swedish twin registry. Occupational Environmental Medicine 58: 52-57.

Pestieau P, Ponthiere G, Sato, M (2006) Longevity and pay-as-you-go pensions. CORE Discussion Paper 54.

Rutkowski J (2006) Labour Market Developments during Economic Transitions. World Bank Policy Research Working Paper 3894.

Vallin J, Caselli G, Surault P (2001) Comportements, styles de vie et facteurs socioculturels de la mortalité. In: G Caselli, J Vallin, G Wunsch (eds.): Démographie : analyse et synthèse, volume III. INED: Paris.

Wilkins R (2004) The extent and consequences of underemployment in Australia. Melbourne Institute of Applied Economics and Social Research Working Paper 16-04.

8 Appendix

Existence of a non-trivial steady-state The case where $\delta > \frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$ can be treated as follows.

Substituting the kk locus within the hh locus allows us to derive an expression giving us h_{t+1} as a function $G(h_t)$:

$$h_{t+1} = Z\left(A(1-\alpha)\left(\left(\frac{A(1-\alpha)(1-\beta)h_t}{\beta+(1-\beta)h_t}\min\left(\left(\frac{[\delta\beta+(1-\beta)h_t]}{(1-\alpha)^{-1}\delta\rho}\right)^{\frac{1}{\rho}},\left(\frac{\beta+(1-\beta)h_t}{\rho}\right)^{\frac{1}{\rho}}\right)\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}\right)^{\psi}$$

$$(\min(\Lambda,\Theta))^{\frac{\psi}{1-\alpha}}\left(\min\left(\frac{\left(\frac{(1-\alpha)[\delta\beta+(1-\beta)h_t]}{\delta\rho}\right)^{\frac{1}{\rho}}}{\left(\frac{\beta+(1-\beta)h_t}{\rho}\right)^{\frac{1}{\rho}}},1\right)\right)^{\phi}$$

where $\Lambda \equiv \left(\frac{(1-\alpha)(\delta\beta+(1-\beta)h_t)}{\delta\rho}\right)^{\frac{-\alpha}{\rho}}$ and $\Theta \equiv \frac{\delta\left(\frac{\beta+(1-\beta)h_t}{\rho}\right)^{\frac{-\alpha}{\rho}}(\beta+(1-\beta)h_t)}{(1-\alpha)(\delta\beta+(1-\beta)h_t)}$.

Under $\delta > \frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$, we know that $\Lambda < \Theta$, and that:

$$\min\left(\left(\frac{(1-\alpha)[\delta\beta+(1-\beta)h_t]}{\delta\rho}\right)^{\frac{1}{\rho}},\left(\frac{\beta+(1-\beta)h_t}{\rho}\right)^{\frac{1}{\rho}}\right) = \left(\frac{(1-\alpha)[\delta\beta+(1-\beta)h_t]}{\delta\rho}\right)^{\frac{1}{\rho}}$$

so that we can rewrite $G(h_t)$ as:

$$h_{t+1} = Z\left([A(1-\alpha)]^{\frac{1}{1-\alpha}}\left(\frac{(1-\beta)h_t}{\beta+(1-\beta)h_t}\left(\frac{[\delta\beta+(1-\beta)h_t]}{(1-\alpha)^{-1}\delta\rho}\right)^{\frac{1}{\rho}}\right)^{\frac{\alpha}{1-\alpha}}(\Lambda)^{\frac{1}{1-\alpha}}\right)^{\psi}\left(\frac{\left(\frac{[\delta\beta+(1-\beta)h_t]}{(1-\alpha)^{-1}\delta\rho}\right)^{\frac{1}{\rho}}}{\left(\frac{\beta+(1-\beta)h_t}{\rho}\right)^{\frac{1}{\rho}}}\right)^{\phi}$$

Given that $G(h_t) \geq 0$, it can be shown that $G(h_t)$ has necessarily a fixed point, that is, a value of h_t such that $G(h_t) = h_t$, if the following 3 conditions are satisfied:

(i) $G(0) = 0$

(ii) $\lim_{h\to 1} G(h_t)/h_t < 1$

(iii) $\lim_{h\to 0} G'(h_t) = +\infty$

Regarding (i), it is easy to check that:

$$G(0) = Z\left([A(1-\alpha)]^{\frac{1}{1-\alpha}}\left(\frac{0}{\beta}\left(\frac{(1-\alpha)[\delta\beta]}{\delta\rho}\right)^{\frac{1}{\rho}}\right)^{\frac{\alpha}{1-\alpha}}(\Lambda)^{\frac{1}{1-\alpha}}\right)^{\psi}\frac{\left(\frac{(1-\alpha)[\delta\beta]}{\delta\rho}\right)^{\frac{\phi}{\rho}}}{\left(\frac{\beta}{\rho}\right)^{\frac{\phi}{\rho}}} = 0$$

Regarding (ii),

$\lim_{h\to 1} G(h_t)/h_t$

$$= Z\left([A(1-\alpha)]^{\frac{1}{1-\alpha}}\left(\frac{(1-\beta)}{\beta+(1-\beta)}\left(\frac{[\delta\beta+(1-\beta)]}{(1-\alpha)^{-1}\delta\rho}\right)^{\frac{1}{\rho}}\right)^{\frac{\alpha}{1-\alpha}}(\Lambda)^{\frac{1}{1-\alpha}}\right)^{\psi}\left(\frac{\left(\frac{[\delta\beta+(1-\beta)]}{(1-\alpha)^{-1}\delta\rho}\right)^{\frac{1}{\rho}}}{\left(\frac{\beta+(1-\beta)}{\rho}\right)^{\frac{1}{\rho}}}\right)^{\phi}$$

so that the constraint imposed is:

$$Z\,[A(1-\alpha)]^{\frac{\psi}{1-\alpha}}(1-\beta)^{\frac{\alpha\psi}{1-\alpha}}\left(\frac{(1-\alpha)[\delta\beta+(1-\beta)]}{\delta}\right)^{\frac{\phi}{\rho}} < 1$$

Regarding (iii), $G(h_t)$ can be rewritten as:

$$G(h_t) = Z\,[A(1-\alpha)]^{\frac{\psi}{1-\alpha}}((1-\beta)h_t)^{\frac{\psi\alpha}{1-\alpha}}(\beta+(1-\beta)h_t)^{\frac{-\psi\alpha\rho-\phi(1-\alpha)}{(1-\alpha)\rho}}\left(\frac{[\delta\beta+(1-\beta)h_t]}{(1-\alpha)^{-1}\delta}\right)^{\frac{\phi}{\rho}}$$

the derivative $G'(h_t)$ is:

$$Z\,[A(1-\alpha)]^{\frac{\psi}{1-\alpha}}\frac{\psi\alpha}{1-\alpha}((1-\beta)h_t)^{\frac{\psi\alpha}{1-\alpha}-1}(1-\beta)\left[(\beta+(1-\beta)h_t)^{\frac{-\psi\alpha\rho-\phi(1-\alpha)}{(1-\alpha)\rho}}\left(\frac{\delta\beta+(1-\beta)h_t}{(1-\alpha)^{-1}\delta}\right)^{\frac{\phi}{\rho}}\right]$$
$$+Z\,[A(1-\alpha)]^{\frac{\psi}{1-\alpha}}((1-\beta)h_t)^{\frac{\psi\alpha}{1-\alpha}}\left[\frac{-\psi\alpha\rho-\phi(1-\alpha)}{(1-\alpha)\rho(1-\beta)^{-1}}(\beta+(1-\beta)h_t)^{\frac{-\psi\alpha\rho-\phi(1-\alpha)}{(1-\alpha)\rho}-1}\left(\frac{\delta\beta+(1-\beta)h_t}{(1-\alpha)^{-1}\delta}\right)^{\frac{\phi}{\rho}}\right]$$
$$+Z\,[A(1-\alpha)]^{\frac{\psi}{1-\alpha}}((1-\beta)h_t)^{\frac{\psi\alpha}{1-\alpha}}\left[(\beta+(1-\beta)h_t)^{\frac{-\psi\alpha\rho-\phi(1-\alpha)}{(1-\alpha)\rho}}\frac{\phi}{\rho}\left(\frac{\delta\beta+(1-\beta)h_t}{(1-\alpha)^{-1}\delta}\right)^{\frac{\phi}{\rho}-1}\frac{(1-\alpha)(1-\beta)}{\delta}\right]$$

Thus, $\lim_{h\to 0} G'(h)$ is:

$$Z\,[A(1-\alpha)]^{\frac{\psi}{1-\alpha}}\frac{\psi\alpha}{1-\alpha}(0)^{\frac{\psi\alpha}{1-\alpha}-1}(1-\beta)\left[(\beta)^{\frac{-\psi\alpha\rho-\phi(1-\alpha)}{(1-\alpha)\rho}}((1-\alpha)\beta)^{\frac{\phi}{\rho}}\right]+0$$

which tends to $+\infty$ when $\frac{\alpha\varphi}{1-\alpha} < 1$.

The three conditions garantee that the function $G(h_t)$ will cross the 45° degree line at least once on the interval $[0,1]$: actually, $G(0)$ is 0, but for higher values of h_t, the $G(h_t)$ curve starts above the 45° line by condition (iii), but must be lower than 1 when h_t tends to 1 by condition (ii) so that it must cross the 45° line somewhere.

The case where $0 < \delta \leq \frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$ can be discussed by following those lines of arguments.

Actually, in that general case, (i) is still true, as:

$$G(0) = Z\left([A(1-\alpha)]^{\frac{1}{1-\alpha}}\left(\frac{0}{\beta}\min\left(\left(\frac{(1-\alpha)[\delta\beta]}{\delta\rho}\right)^{\frac{1}{\rho}},\left(\frac{\beta}{\rho}\right)^{\frac{1}{\rho}}\right)\right)^{\frac{\alpha}{1-\alpha}}\right)^{\psi}$$
$$\left(\min\left(\left(\frac{(1-\alpha)(\delta\beta)}{\delta\rho}\right)^{\frac{-\alpha}{\rho}},\frac{\left(\frac{\beta}{\rho}\right)^{\frac{-\alpha}{\rho}}}{(1-\alpha)}\right)\right)^{\frac{\psi}{1-\alpha}}\left(\min\left(\frac{\left(\frac{(1-\alpha)[\delta\beta]}{\delta\rho}\right)^{\frac{1}{\rho}}}{\left(\frac{\beta}{\rho}\right)^{\frac{1}{\rho}}},1\right)\right)^{\phi} = 0$$

Regarding (ii), one should notice that, if $\delta \leq \frac{(1-\alpha)(1-\beta)}{\alpha\beta+(1-\beta)}$, the economy cannot remain in underemployment when h_t tends to 1. Therefore, the condition under which $\lim_{h_t\to 1} G(h_t)/h_t < 1$ becomes:

$$Z\,[A(1-\alpha)]^{\frac{\psi}{1-\alpha}}(1-\beta)^{\frac{\alpha\psi}{1-\alpha}}\left(\frac{(1-\alpha)[\delta\beta+(1-\beta)]}{\delta}\right)^{\frac{-\psi}{1-\alpha}} < 1$$

Regarding (iii), one should notice that, when h_t tends to 0, the economy must, given $\delta > 0$, tend to excess labour supply, so that the above condition $\frac{\alpha\psi}{1-\alpha} < 1$ must remain true.

The case where δ is zero is trivial. Under $\delta = 0$, the negotiated wage is zero, so that workers will not be able to consume nor to save anything. Hence, given the full depreciation of capital, the capital stock at the next period will be zero, so that the production can no longer take place, whatever initial conditions are.

Uniqueness If the steady-state exhibits no underemployment, $G(h_t)$ becomes:

$$h_{t+1} = G(h_t) = Z\left[A(1-\alpha)\right]^{\frac{\psi}{1-\alpha}} \left(\frac{\delta}{1-\alpha}\right)^{\frac{\psi}{1-\alpha}} \left[(1-\beta)h_t\right]^{\frac{\alpha\psi}{1-\alpha}} \left(\beta + (1-\beta)h_t\right)^{\psi} (\delta\beta + (1-\beta)h_t)^{\frac{-\psi}{1-\alpha}}$$

However, it follows from this that the derivative of $G(h_t)$ at such a steady-state must be non-positive. Indeed, $G'(h_t)$ can be rewritten as the product:

$$\left[Z\left[A(1-\alpha)\right]^{\frac{\psi}{1-\alpha}} \left(\frac{\delta}{1-\alpha}\right)^{\frac{\psi}{1-\alpha}} \left[(1-\beta)h_t\right]^{\frac{\alpha\psi}{1-\alpha}} \left(\beta + (1-\beta)h_t\right)^{\psi} \left(\delta\beta + (1-\beta)h_t\right)^{\frac{-\psi}{1-\alpha}} (1-\beta)\right]$$
$$\left[\frac{\alpha\psi}{1-\alpha}\left[(1-\beta)h_t\right]^{-1} + \psi\left(\beta + (1-\beta)h_t\right)^{-1} + \left(\frac{-\psi}{1-\alpha}\right)\left(\delta\beta + (1-\beta)h_t\right)^{-1}\right]$$

whose first factor is positive, whereas the second factor is necessarily non-positive.

Actually, a non-positive second factor is equivalent to:

$$\frac{\alpha}{1-\alpha}\frac{1}{(1-\beta)h} + \frac{1}{\beta+(1-\beta)h} \leq \frac{1}{1-\alpha}\frac{1}{\delta\beta+(1-\beta)h}$$

which, in terms of δ, is equivalent to: $\delta \leq \frac{(1-\beta)(1-\alpha)h}{\alpha\beta+h(1-\beta)}$.

But there is no underemployment here, so that $\delta \leq \frac{(1-\beta)(1-\alpha)h}{\alpha\beta+h(1-\beta)}$ is necessarily satisfied.

Hence, under that constraint, $G'(h) \leq 0$. In other words, the slope of $G(h)$ in the full employment area (i.e. on the right of the vertical dotted line) is non-positive. It follows from that inequality that the non-trivial steady-state must here be unique. Actually, given that $G(h)$ is decreasing with h for $h > h^*$, it is hard to see how $G(h)$

could cross the 45° line another time on the interval $[h^*, 1]$, so that the non-trivial steady-state must be unique.

9 Tables

Table 1: Comparative statics.

	Steady-state (full employment)			Steady-state (underemployment)		
Parameters	k^*	h^*	e^*	k^*	h^*	e^*
δ	?	?	?	?	?	-
ψ	+	+	=	+	+	+
ϕ	=	=	=	-	-	-
Z	+	+	=	+	+	+
β	-	?	?	-	-	-
ρ	=	=	=	=	=	=

10 Figures

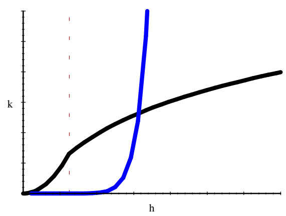

Figure 1: Full-employment SS

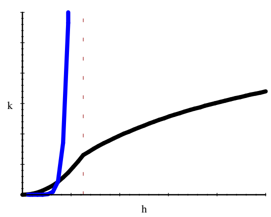

Fig. 2: Underemployment SS

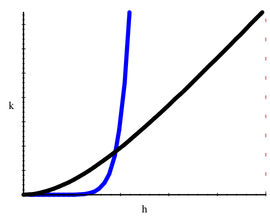

Fig. 3: Underemployment SS

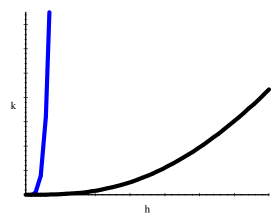

Fig. 4: No equilibrium

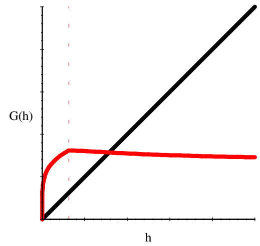

Fig. 5: Full-employment equilibrium

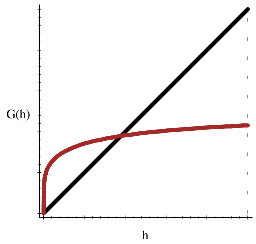

Fig. 6: Unique underempl. SS

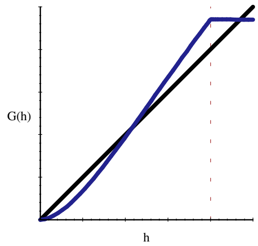

Fig. 7: Two SS of distinct kinds

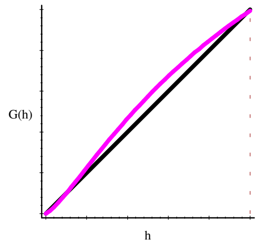

Fig. 8: Two underempl. SS

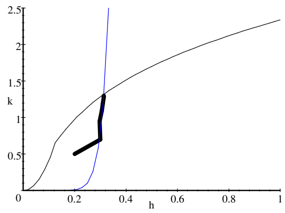

Fig. 1bis: Stability of the SS

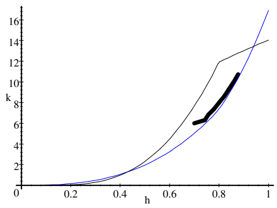

Figure 7bis: Stability of high SS

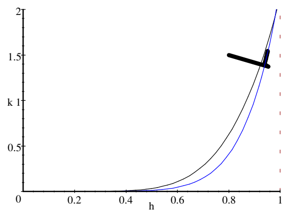

Figure 8bis: Stability of high SS

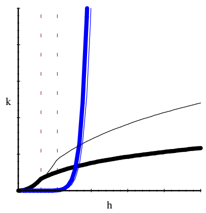

Figure 9a: A rise of δ

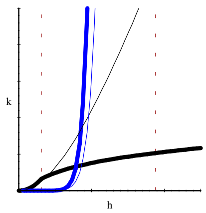

Figure 9b: A rise of δ

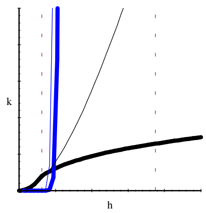

Figure 9c: A rise in δ

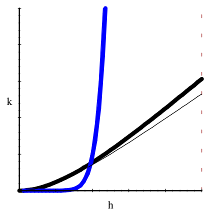

Figure 9d: A rise in δ

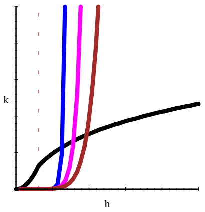

Fig. 10a: A rise in ψ

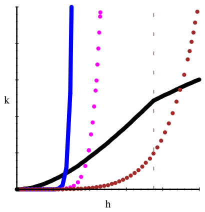

Fig. 10b: A rise in ψ

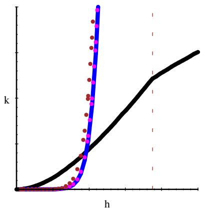

Fig. 10c: A rise in ϕ

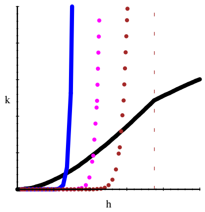

Fig. 10d: A rise in Z

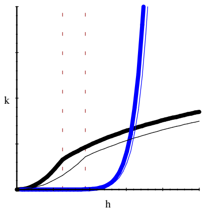

Fig. 11a: A rise in β

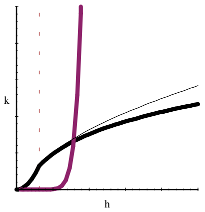

Fig. 11b: A fall in ρ

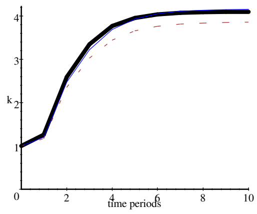

Fig. 12a: Capital

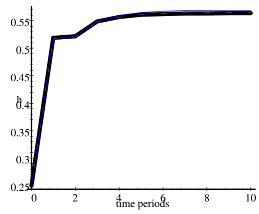

Fig. 12b: Longevity

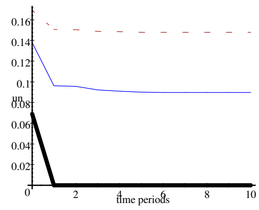

Fig. 12c: Underemployment

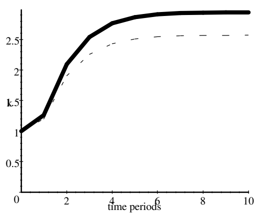

Fig. 13a: Capital

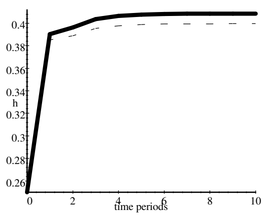

Fig. 13b: Longevity

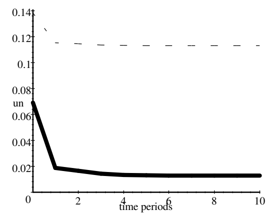

Fig. 13c: Underemployment

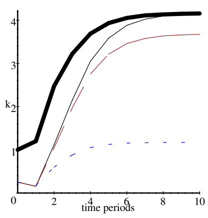

Fig. 14a: Capital

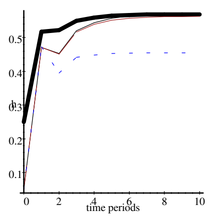

Fig. 14b: Longevity

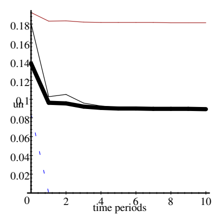

Fig. 14c: Underempl.